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SOME JUDGMENTS ON THE SIZE OF FAMILIAR OBJECTS.

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Many curious errors in the reproduction of one magnitude in terms of another have long been known. How many city folks can tell in feet and inches the height of an average horse? How certainly the non-professional individual, who can estimate this object closely, will measure the height on himself! The weight of a carriage, the number of sheep, hogs or cattle in a drove, the height of a tree or a steeple, the apparent size of the moon; in all these problems the personal variation would probably be great, and of course would be much greater if the estimate were made on a memory image instead of from an immediate perception. How many persons who have seen the original of the Sistine Madonna can tell with much confidence whether the virgin is "life size"; or better, whether the figure is of average size, or is larger or smaller than the average woman? In nearly all the above cases the individual variations would tend to balance each other, so that a very great constant error would not be looked for.

In many familiar instances the constant error is considerable, as, for example, in attempting to indicate on the wall the distance from the floor which corresponds to the height of a silk hat, a doubling of the actual height is not infrequent. Distances of a mile or more in the city are generally underestimated and in the country overestimated; while in certain obvious cases this tendency would be increased and in other cases might be reversed.

Few persons would fail to underestimate the length of a horse's head, especially if asked to mark it off on the blackboard or, as is the custom, on a barrel. Anyone unfamiliar with such instances may learn a valuable lesson in self-distrust by placing three silver dollars in a line just touching each other and then drawing away the middle dollar far enough to make the distance from it to each of the other dollars equal to the distance between the outer edges of the other two. Besides the usual explanation of illusions we have here to deal with the very uncertain factor of multiples of an unusual unit.

In these and other cases of immediate judgment of space relations, there are two quite distinct sources of error. One arises from the common illusion of space perception when the things to be compared are both present to the senses. No one ever *imagined any* illusions of the senses. So also no amount of reasoning will convince the practical, uninitiated man that his illusive perceptions are not true perceptions of real things. He must refer these stimuli to some objective standard which commands his belief, or he must neutralize the disturbing factors.

The other source of error is the neglect of uninteresting and useless details in complex presentations. The only interest we have in the height of a hat is relative to other hats. Height in inches, or as compared with that of a wall, is almost the least interesting fact regarding a hat. This knowledge is also useless. The abstraction of these commonly unused details from our memory pictures of familiar objects has been employed for illustrative purposes by psychologists of all times. As far as I know there has been no attempt at a quantitative analysis of the facts.

In connection with classes in the study of children it has been my custom for several years to spend some time in the public schools of Lincoln, Nebraska. One result of this work for the year 1893 is the data upon which this paper is based. The repetition of the exercise with successive classes in psychology supplements this material in a satisfactory manner.

I wished to know how accurately children and adults can reproduce the size of objects with which they are familiar, but which for the time are not present to the senses. A few other points were incidentally included in the investigation. In selecting objects there was encountered the difficulty of finding common things with a definite and constant size, which should be unknown in terms ordinarily used for measuring size. To ask for the dimensions of a common brick would be useless; for actual bricks differ considerably in size, and the standard size of a brick is known to many chil-

dren. Value is perhaps as loosely associated with size as any other concept, and our coins and bills are quite constant in form and dimensions. Money is also common enough to furnish a memory image to nearly all children who could be expected to give reliable results on a question involving the description of a memory picture.

I selected the following coins: dollar, half dollar, quarter dollar, dime and nickel; also the five-dollar bill. Few children have a very definite standard of length at command. Instead of asking for dimensions, the problem was to reproduce on paper the size and form of the object. The paper used was 14 x 9 inches in size. Altogether fourteen questions were given. The teachers were nearly all interested, and they aided not a little in securing honest, intelligent and prompt results.

The procedure was as follows: The pupils of a given room all worked at the same time. Each child was given a sheet of paper, and told to write his name, age and grade in the upper left-hand corner as the paper lay on the desk with the long side parallel to the long side of the desk. Just below his name near the left margin of the paper he was told to draw a circle the size of a silver dollar. A diagram of the paper was put on the blackboard, and the position of the dollar was indicated by a small cross. It was emphasized that size was more important than roundness. The pupils were told to correct their first attempt by erasure or by striking out parts of the curve whenever the circle seemed too large or too small. About one minute was given to this and to each of the other problems, though more time was given when desired. Just at the right of the dollar they were asked to draw a half dollar; at the right of that a quarter dollar, then a dime and a nickel. Only one problem was given at a time, yet the children were not prohibited from making corrections in the figures already drawn. (I do not think that such changes were made by *more* than five per cent. of the children.) The fractional coins were called for as above, and each time the number of cents was announced, so that the children could not be in doubt as to what was wanted.

After drawing the nickel the children were asked to draw a square equal in area to the sum of the areas of all the circles drawn. When necessary this was explained, and in all cases the statement was put in two or three forms. Just below the coins was drawn a figure equal in size and similar in form to a five-dollar bill (or to a one or two dollar bill). In the lower right-hand corner of the paper a circle three inches in diameter was drawn; in the lower left-hand corner a square one inch on a side, and between these an equilateral triangle

equal in area to the sum of the areas of the triangle and square. The length, width and diagonal of the paper in inches were then required. The equilateral triangle was not drawn by the children of the fourth grade.

The entire time in any one room seldom exceeded twenty minutes and never occupied less than fifteen minutes. As much personal supervision as possible was given by the teachers and myself, though, of course, without any suggestion as to the points involved in the investigation. I personally conducted the work in every room, and was assisted by the teacher in charge, and often by my pupils. All measurements of results and nearly all computations were made by myself. No results are omitted from the tables. The entire work in the schools was completed in as short a time as possible. It is not probable that any material difference in results was produced by practice or anticipation. Since I examined only two grades—the fourth and the eighth,—it was possible to complete the work in each building in one-half day. Assistance from previous information and from comparison with standards was thus reduced to a minimum. The results of the various problems are by no means of equal interest. It seems best, however, to present the facts obtained separately and in the order of the original problems. A table and summary of general conclusions may then follow.

I.

Reproduction of the Size of a Silver Dollar.

All circles were measured on two diameters parallel with the edges of the paper, unless a perceptibly better adjustment presented itself. Measurements were recorded to the nearest millimeter. But, since the two diameters were taken in immediate succession, it was possible to attain even greater accuracy by attention to compensation. This was made especially easy by the habit of measuring both diameters before recording the first.

The tables are, for the most part, self-explanatory. A few words concerning the methods of obtaining the average and the average variation may be useful. It is evident that the arithmetical mean or average of all judgments in a given class, when compared with the actual dimensions of the object, will indicate the direction and amount of the constant error. In most cases the constant error is so small that it did not seem necessary or even wise to emphasize it by separation from the average, since a larger number of experiments

might possibly have reversed it. In all cases it may easily be obtained directly from the tables.

The average variation is obtained, as usual, by subtracting each judgment from the average of all in a given class, adding the remainders without regarding algebraic signs, and dividing by the number of judgments. It is simply the average variation of separate judgments from the arithmetical mean of all judgments. The average variation is a fair measure of the relative appreciation of differences on the part of the pupils in various classes. It must not be confused with the average error, which in a somewhat similar manner measures the accuracy of judgment, but in the present investigation is less interesting.

TABLE I.

Silver Dollar = 37.8 mm.

FOURTH GRADE.

AGE.	No. of Persons.	MALES.		No. of Persons.	FEMALES.	
		Average.	Av. Variation.		Average.	Av. Variation.
9	46	37.2	6.2	60	36.8	5.3
10	63	38.0	5.3	63	38.0	6.0
11	35	35.8	3.9	53	38.4	5.7
12	33	37.4	5.9	28	38.5	6.0
13	19	37.9	6.0	16	36.3	7.1
Average,		37.3	5.5		37.6	6.0
		EIGHTH GRADE.				
13	25	37.2	3.1	41	39.2	4.0
14	31	39.5	3.4	40	38.7	4.0
15	29	38.2	3.4	47	38.3	3.1
16	21	38.9	3.7	42	39.5	4.3
Average,		38.5	3.4		38.9	3.9
YEAR.		PSYCHOLOGY CLASS.				
1893	22	39.3	2.9	22	39.4	4.6
1894	30	41.2	6.0	42	41.4	4.6
1895	43	38.4	4.0	37	39.5	3.5
1896	53	39.1	4.3	33	38.5	3.2
1897	64	38.9	4.0	48	39.5	3.3
Average,		39.4	4.2		39.7	3.8

Table I shows that no appreciable constant error is made by children of the fourth grade in estimating the size of a silver dollar. There is no difference in the results that can be referred to difference in age. There is, however, a distinct though slight increase in the size of the circles as made by the eighth grade children. This increase is more marked in the case of the girls. The more advanced the children, the larger the dollar seems and the greater the error is. The same conclusion is also indicated by the results from university students. Indeed the tendency is even more strongly marked, as is shown by the lower part of the table. The fourth grade children, then, make the dollar about the proper size, and a decided tendency to overestimate its size is shown by the eighth grade pupils, and still more so by advanced university students. Although this tendency is unmistakable, the constant error is, on the whole, rather insignificant in itself. In connection with other measurements it has a deeper meaning.

In the columns marked average variation we observe considerable uniformity within each class and rather large differences between the classes. As might be expected the immature pupils of the fourth grade show the greatest variations from each other. Let us ascribe this to their ignorance. We shall then have to account for the contrary results of the eighth grade and university students by the development of individuality in the university classes.

We should hardly have expected the difference between boys and girls which this table shows. Yet it will be seen that the same result holds good throughout the present investigation, that is, *girls differ from each other more than boys do in their ideas of the size of such objects as are here studied.* If this fact shall be proved true generally, it may become of considerable pedagogic importance. Of course in this connection it merely means that the girls examined were in general inferior to the boys in ability to reproduce the sizes of the objects asked for. The apparent equality of the sexes in the general averages means that the sexes have practically the same constant errors in estimating the size of a dollar.

The university students are not as well able as children of the advanced grades to estimate the size of a silver dollar. Moreover they differ more from each other than do the children of the eighth grade. Beyond a certain very elementary stage general knowledge is not power in accurately reproducing the space dimensions now under consideration. The average variations of the men for the first two years seem abnormal. I am entirely unable to account for the peculiar

results exhibited. The men of the class of '94 seem to differ from each other extremely in all these tables. This is partly due to the presence in that class of three or four *large-hearted* fellows who do and see all things in a generous way. They also raised appreciably the average size of all the figures. It is not believed that mutual influence played any part in their judgments. There happened to be also two or three women in that same class who had exalted ideas of the size of coins. If these persons had been closely associated with each other,

II.

Reproduction of the Size of a Silver Half Dollar.

TABLE II.

Half Dollar = 30.6 mm.

FOURTH GRADE.

AGE.	No. of Persons.	MALES.		No. of Persons.	FEMALES.	
		Average.	Av. Variation.		Average.	Av. Variation,
9	46	28.8	4.7	60	29.6	5.1
10	63	30.9	4.4	63	30.4	5.1
11	35	29.4	4.3	53	30.4	5.1
12	33	29.9	4.3	28	30.9	6.0
13	19	30.7	4.5	16	29.6	6.2
Average,		29.9	4.4		30.2	5.5
		EIGHTH GRADE.				
13	25	30.8	2.2	41	32.0	3.2
14	31	31.5	2.7	40	31.1	3.6
15	29	30.2	3.1	47	31.0	2.9
16	21	30.1	2.7	42	32.0	3.7
Average,		30.9	2.7		31.5	3.3
YEAR.		PSYCHOLOGY CLASS.				
1893	22	31.9	3.2	22	32.1	3.5
1894	30	33.5	4.5	42	34.1	3.5
1895	43	31.2	3.5	37	32.4	3.0
1896	53	31.9	3.5	33	31.7	3.2
1897	64	31.2	3.3	48	31.8	3.0
Average,		31.9	3.6		32.4	3.2

I should accept the result as an indication of collusion. They were not intimate, and in nearly every case were the last persons of the whole class to suspect of any unfairness. In later classes nearly as extreme judgments occur, but in all other classes such errors are proportionally fewer.

Almost exactly the same characteristics are to be observed of the half dollar that were noticed in the case of the dollar, especially the near approach of the averages to the actual size of the coin, and the smaller averages of the fourth grade pupils, in this table somewhat under the actual size. We note also the approximate equality of constant errors between the sexes, the slight difference resulting from the greater size of the circles made by the girls. As in Table I there is seen little effect due to age within the same grade, but we observe the clearly marked influence of school training in the larger differences between the lower and higher grades.

The average variation also shows the same relative results as in case of the dollar. It is, however, noticeably less, and perhaps the difference in favor of the boys in the fourth and eighth grades is even greater, though as in Table I the women of the psychology class show a smaller average variation than the men. I would call especial attention to the regularity of this average variation in case of the boys of the fourth grade. For the small number of boys and their small progress in school knowledge this regularity forms an excellent example of the presence of law and of the possibilities of such investigations.

We see in this table the same low average variation of the boys of the eighth grade, which doubtless has a constant cause. It is possible that this means, in connection with the greater variations for lower and higher grades, that, at this period of life and stage of advancement, the advantage of knowledge and experience is at a maximum, and that individuality has not yet strongly manifested itself. But we must beware of generalization from so few facts. This warning applies especially to the apparently clear indication that girls are brighter than boys, inasmuch as the effect of general training is to increase the size of the figure, and the tables show that girls regularly make larger figures than those made by boys.

III.

Reproduction of the Size of a Silver Quarter Dollar.

In reproducing the quarter dollar the constant error almost disappears. The lower grades make the circle just a little too small and the university classes make it a little too large.

The relations of the averages to each other are almost exactly the same in kind as in Tables I and II. In amount the differences between the various classes are less than in the preced-

TABLE III.

Quarter Dollar = 24 mm.

AGE.	No. of Persons.	MALES. Average.	MALES. Av. Variation.	No. of Persons.	FEMALES. Average.	FEMALES. Av. Variation.
FOURTH GRADE.						
9	46	22.5	3.3	60	23.5	3.7
10	63	23.6	3.4	63	23.6	3.6
11	35	23.1	4.1	53	23.7	4.0
12	33	22.5	3.9	28	25.3	4.5
13	19	23.3	3.0	16	23.9	4.6
Average,		23.0	3.5		24.0	4.1
EIGHTH GRADE.						
13	25	23.3	2.0	41	24.7	2.4
14	31	23.4	2.5	40	23.8	2.5
15	29	23.1	2.7	47	24.4	2.3
16	21	22.9	2.9	42	24.7	3.1
Average,		23.2	2.5		24.4	2.6
YEAR.		**PSYCHOLOGY CLASS.**				
1893	22	24.2	2.2	22	25.1	2.2
1894	29	26.2	3.4	42	26.4	2.4
1895	43	23.8	2.8	37	24.6	2.3
1896	53	24.6	2.6	33	24.6	2.8
1897	64	24.4	2.7	48	24.4	2.3
Average,		24.6	2.7		25.0	2.4

ing tables. The uniformity of the several groups within each large division is broken only by the class of '94 of university students, and by both men and women of this class.

The average variation has decreased considerably, even in relation to the size of the object. The relative difference between boys and girls has also decreased, and, as in Tables I and II, the men of the psychology class show a greater average variation than the women.

IV.

The Dime.

Since the two following tables of results (for the dime and nickel) show the reverse tendency, *i. e.*, an *underestimation* of the size, the question arises as to the effect of the first made circles on those coming after. It happens that the quarter was preceded and also followed by two other judgments of coins, and that those preceding were overestimated and those following were underestimated, while the quarter itself was judged about right. The discussion of this question is postponed until the evidence is all before us.

TABLE IV.

Dime = 19 mm.

FOURTH GRADE.

AGE.	No. of Persons.	MALES.		No. of Persons.	FEMALES.	
		Average.	Av. Variation.		Average.	Av. Variation.
9	46	13.8	2.3	60	14.9	2.2
10	63	14.7	2.2	63	14.6	2.3
11	35	14.9	2.5	53	15.1	2.4
12	33	14.6	2.3	28	15.3	1.9
13	19	14.3	2.1	16	14.6	2.0
Average,		14.5	2.3		14.9	2.2
		EIGHTH GRADE.				
13	25	15.2	1.7	41	15.5	1.6
14	31	14.9	1.7	40	14.5	1.8
15	29	14.5	1.4	47	15.7	1.6
16	21	14.9	1.6	42	15.0	1.6
Average,		14.9	1.6		15.2	1.7
YEAR.		PSYCHOLOGY CLASS.				
1893	22	15.0	1.8	22	15.5	1.3
1894	30	16.9	2.4	42	16.1	1.8
1895	43	15.5	1.9	37	15.3	1.4
1896	53	15.5	2.1	33	15.5	1.6
1897	64	15.6	2.0	48	15.7	1.5
Average,		15.7	2.0		15.6	1.5

In estimating the size of a dime the observers for the first time lose their bearings. The constant error here claims our immediate attention by its enormous and unexpected size. The discrepancy between the real and the represented area will provoke the speculative mind to all sorts of explanations. The first naturally to suggest itself is the fact that, beginning with a larger circle, the dollar, the others are made in relation to those preceding, and hence the later ones especially are influenced by the general law of relativity, that alongside of large objects the small seems smaller than it really is. However this may be, I have convinced myself that the tendency to make a silver ten-cent piece too small does not depend upon the presence or suggestion from the outside of a large circle. Enough experiments were made later to indicate a decided underestimation of the size of a dime without conscious reference to other coins. Nevertheless it is doubtless true that the coin series, or a part of it, is responsible for this great constant error. It is also probable that this tendency has been exaggerated in the present experiments by means of the attention given to the large circles before attempting to draw the dime. My supplementary experiments were not numerous enough to warrant a quantitative comparison.

A further *a priori* consideration may have contributed to the magnitude of the constant error, namely, the fact that a dime, though smaller than a nickel, has a greater value. Doubtless such an apparent contradiction is likely to be over influential, and as the value is clear the difference is transferred to the size. It is a double application of the principle of relativity. That is, as compared with the dollar in size the dime would be underestimated; as compared with the nickel in value and size it would also appear smaller than it really is (just as a small great man seems smaller than a small unknown man). The same processes would also tend to make the nickel larger than it would otherwise be, and I am not sure but this is the case.

In other respects Table IV shows the same peculiarities as the tables already presented. The average variation is of course smaller and is also more constant within each of the various groups. For the first time it becomes less than the constant error.

V.

The Nickel.

In trying to draw a circle the size of a nickel the children of the lower grades make nearly as large constant errors as when reproducing the size of a dime. The university stu-

dents, however, are much more accurate in this problem than they were in that of the smaller circle. In other respects Table V shows the same tendencies and characteristics as the preceding tables.

TABLE V.

Nickel = 21 mm.

FOURTH GRADE.

AGE.	No. of Persons.	MALES.		No. of Persons.	FEMALES.	
		Average.	Av. Variation.		Average.	Av. Variation.
9	46	16.7	2.3	60	17.2	2.7
10	63	17.3	2.3	63	17.2	2.7
11	35	17.5	2.2	53	17.2	2.4
12	33	16.8	2.0	28	17.5	2.5
13	19	16.8	2.2	16	16.8	2.7
Average,		17.0	2.2		17.2	2.6
		EIGHTH GRADE.				
13	24	18.1	2.2	41	18.4	1.9
14	31	18.1	2.1	40	17.5	2.2
15	29	17.6	1.9	47	18.5	2.0
16	21	18.4	1.8	42	17.5	1.9
Average,		18.1	2.0		18.0	2.0
YEAR.		PSYCHOLOGY CLASS.				
1893	22	18.2	2.3	22	19.1	1.9
1894	30	19.9	2.7	42	20.0	2.2
1895	43	18.4	2.6	37	18.7	1.8
1896	53	18.8	2.0	33	18.9	1.7
1897	64	19.3	2.4	48	18.9	1.8
Average,		18.9	2.4		19.1	1.9

These five tables of judgments on the size of common coins may be studied together. They show that the less advanced children make all figures smaller than those made by the more advanced persons. Within any given class age does not seem to produce as great effect as does the degree of advancement in knowledge. That is, the children of nine years in the fourth grade do not, as a rule, make circles much different from those made by children thirteen years of age in the same grade. On the other hand thirteen-year old chil-

dren of the eighth grade usually differ considerably from fourth grade children of the same age. In all cases except that of the dime as drawn by university students, the girls make larger circles than the boys. The variation of the girls is in most cases greater than that of the boys. There is a slight tendency in both sexes to overestimate the size of the larger circles, and, proportionally, a much greater tendency to underestimate the size of the small circles.

In the average variation we observe much difference between elementary and advanced students except in the results for the two smaller circles. In the estimates of the dime and nickel, scholastic knowledge seems to contribute little toward a diminution of the differences between individuals. This is especially true in regard to the nickel.

VI.

Squaring the Circles.

After finishing their circles the pupils were asked to observe them in a careful manner, and to draw a square equal in area to the sum of the areas of all the circles. It is evident that this is a different kind of problem from those they had been dealing with. No memory image is necessary for the performance of the task now under consideration. No geometrical calculation would have assisted their judgment, for they had no accurate measurements, and the time at their disposal would not admit of even a hurried estimate based on a guess at dimensions and a calculation of areas. I believe, therefore, that the judgments were, for the most part, just as I desired them to be, off-hand estimates of the areas of the several circles in terms of a square.

The nearest possible approach to a systematic calculation would be in the rapid determination of the area of each circle in square inches and the summation of these results. The representation of a given number of square inches in the form of a square would then be a simple matter. It is not probable, however, that half even of the most advanced students made use of any similar methods, while only exceptional cases in the lower grades would have found any help from indirect methods. It is safe to say that in most cases the processes were unconscious, and that the comparison seemed to be immediate between the circles and their equivalent square. The square is therefore subject to two sources of error, inasmuch as the circles may have been wrongly drawn, and the translation and summation of the circles into a square may have been wrongly done.

TABLE VI.

Square, 54.2 mm. on a side.

FOURTH GRADE.

AGE.	No. of Persons.	MALES.		No. of Persons.	FEMALES.	
		Average.	Av. Variation.		Average.	Av. Variation.
9	45	80.1	23.5	60	82.4	20.9
10	63	76.7	14.7	63	85.1	24.5
11	34	77.6	21.4	53	81.8	19.4
12	33	85.6	21.2	28	83.1	21.4
13	18	83.9	25.4	16	84.0	21.0
Average,		80.8	21.3		83.3	21.4
Should be		51.0			51.8	
		EIGHTH GRADE.				
13	25	69.2	13.4	41	69.9	15.3
14	31	64.4	8.9	40	69.3	14.5
15	29	63.0	9.5	47	66.4	12.7
16	21	60.8	12.0	42	79.3	17.0
Average,		64.4	11.0		71.2	
Should be		52.6			53.6	14.9
YEAR.		PSYCHOLOGY CLASS.				
1893	22	60.0	7.4	22	60.6	8.0
1894	30	63.7	10.2	42	70.2	11.9
1895	43	57.8	7.5	37	63.7	8.0
1896	53	58.8	8.1	33	64.1	9.1
1897	64	59.8	9.8	48	63.4	9.9
Average,		60.0	8.6		64.4	9.4
Should be		54.3			55.0	

The size of a square equivalent in area to the sum of the areas of the five coins is given at the head of the table. The size of a square equivalent to the average circles of each grade of pupils is given at the bottom of each division of the table. That is, the boys of the fourth grade should have made the square 51 mm. on a side instead of nearly 81 mm. If the circles had been made without constant error, each side of the square should have been about 54.2 mm.

The girls of the fourth grade make the largest squares (83.3 mm.). This is about two and one-half times the re-

quired area, and would be still thirty per cent. too large if we subtracted the entire average variation from the side of the average square. The boys of the fourth grade make a square but little smaller than that made by the girls.

Nearly half of the excess in the size of the square is lost in the work of the eighth grade pupils, though the difference between boys and girls is much greater than in the fourth grade. The average variation in this grade is but a little less than the constant error. There is much greater difference in the average variations of this entire table than of any other thus far considered. The problem is of course more complex, and hence individuals have greater latitude. It is rather surprising that the sub-groups within the same grade should differ so greatly from each other, especially in average variation.

The university students still further decrease the constant error until it is only about two-thirds as large as the average variation of the members of the class.

It will be noticed that the circles of the lower grades are smaller than they should be, and hence the square which would contain the actual circles before their eyes is smaller than that indicated at the top of the table. The boys of the fourth grade lose about 3 mm. on each side of the square in this manner, the girls about 2 mm. In the eighth grade the loss is smaller. The girls of this grade make the circles on the average almost exactly the right size. The undersize of the dime and nickel is made up by the overestimation of the dollar and half dollar. The university students have on the whole slightly overestimated the size of the coins, the men almost not at all, but the women have done so to an appreciable degree.

There is, after all, very little difference in the areas of the circles drawn by the several grades of students. The immense difference between the areas of the squares as drawn by fourth grade pupils and university students is due to overestimation of the circles by the lower grades—to the inability of the more immature children to square the circles. It will be thought that the children misunderstood the problem. It is true that some few may have believed, after all warning and explanation, that I desired a square which would *contain* the circles unbroken. I believe, however, that very few persons failed to grasp the problem of equivalent areas.

VII and VIII.

The Size of a Five-Dollar Bill.

Tables VII and VIII are most interesting. It was required to draw a rectangle equal in size and similar in form to a five-

dollar bill (or a one-dollar bill). This problem presented great difficulty, if we judge by results, and yet the subjects seemed to have much confidence in the accuracy of their work. The proportion in the form was well maintained throughout the papers, thus indicating clearly that the persons concerned knew what they were trying to do. We must also believe, in this case at least, that the larger size of the bill, as made by advanced students, is due chiefly to more accurate power of imaging.

The constant error is so unexpectedly large, and the difference between fourth grade pupils and university students is so great, that the factors which might account for the divergences in case of the circles will not be accepted here. The slight influence of age as compared with that of grade is seen in these two tables more clearly than I remember to have seen it elsewhere. But it may be said that this whole paper is a striking illustration of the same fact, and it is just about time for some evidence of this kind to be forthcoming, since there has been a tendency to affirm the opposite on the evidence furnished by several experimental researches. It is perhaps a fault of the psychological monograph to generalize beyond its province. It is clear that, as far as these tables show, age has little to do with accuracy of reproduction of space relations, while the degree of academic advancement has much to do with the matter. We must not conclude, however, that greater extremes of age, or other problems in space relations, would show the same kind of results. It is at least unwise to make any generalization on this point without more facts than are at present in our possession.

It seems almost incredible that over 400 children, who have had four years' training in our public schools, should, on the average, think of a dollar bill as barely half its actual size. Has instruction in drawing called away attention from magnitude in order to emphasize form? There would seem to be at least great neglect in the recognition of absolute dimensions, perhaps not unlike the well-known inability of most musicians to judge the absolute pitch of tones.

Reference to the original tables shows that only four fourth grade children out of 415 made the bill long enough, and only thirteen made it wide enough. In the eighth grade, out of 275 pupils, only six made the bill long enough, and only sixteen made it wide enough. Of 394 university students twenty-four made it long enough, and fifty-eight wide enough.

Considering the size of the constant error in Table VII, the variation among individuals is but little. In the fourth grade it amounts to less than one-third of the constant error, and decreases in the higher classes, though not nearly as fast as

TABLE VII.

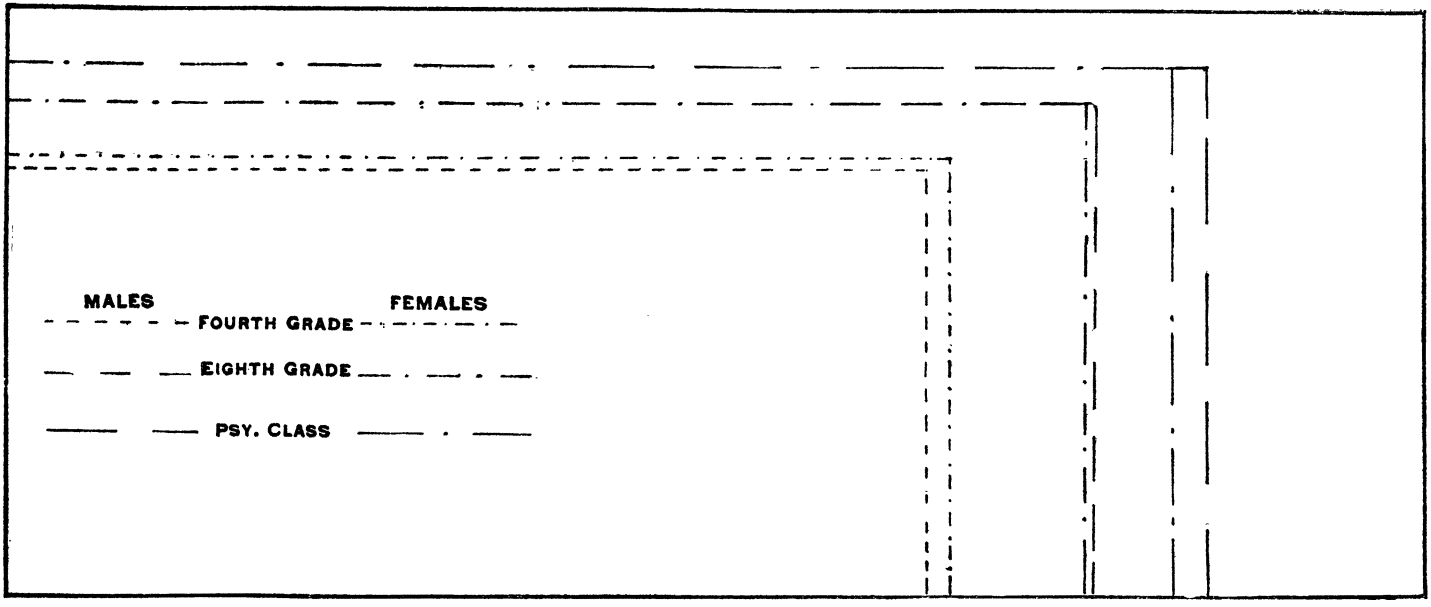
Length of Bill = 186.5 mm.

FOURTH GRADE.

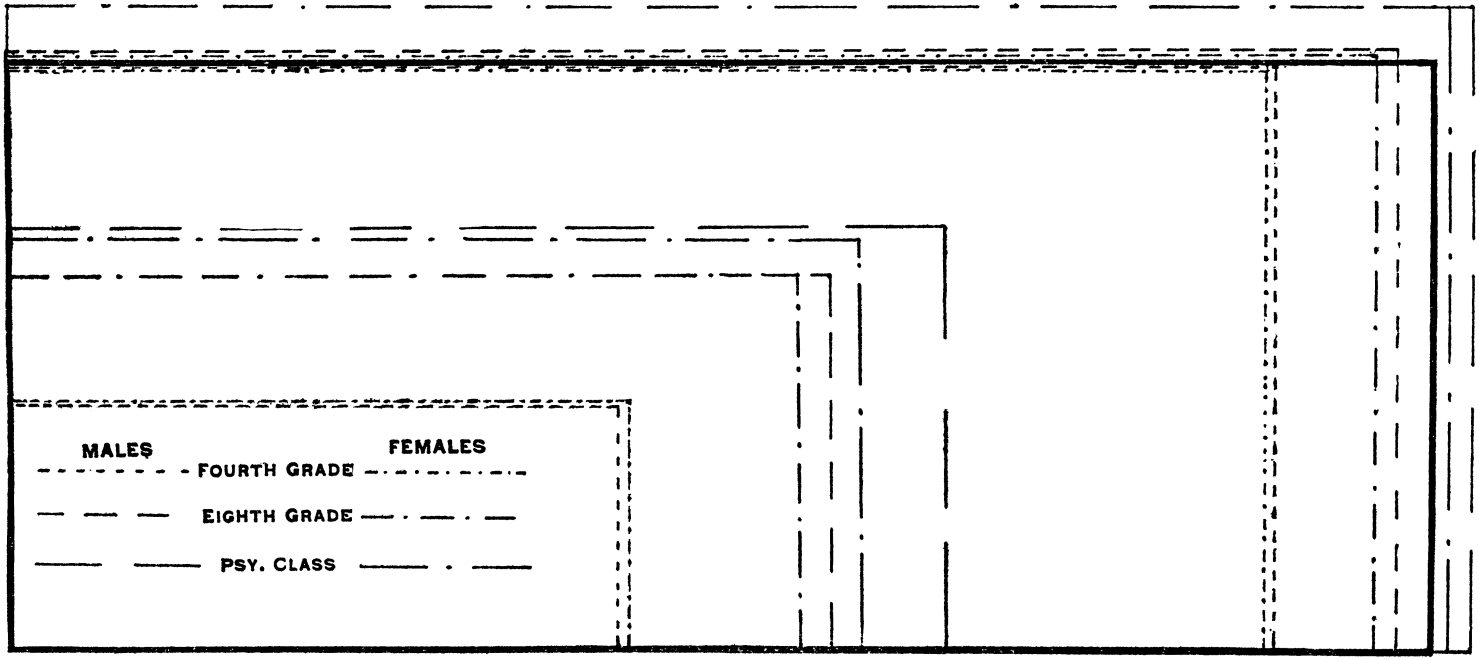
AGE.	No. of Persons.	MALES.		No. of Persons.	FEMALES.	
		Average.	Av. Variation.		Average.	Av. Variation.
9	46	120.6	19.8	60	119.5	17.1
10	63	123.6	20.5	63	122.8	17.4
11	35	123.8	16.6	53	129.6	17.4
12	33	117.2	21.6	27	129.6	16.4
13	19	119.3	18.4	16	118.3	23.8
Average,		120.9	19.4		124.0	18.4
		EIGHTH GRADE.				
13	25	146.7	14.2	41	144.9	16.8
14	31	140.1	15.4	40	138.0	17.1
15	29	137.1	18.3	47	140.0	17.5
16	21	146.2	17.7	41	143.4	15.4
Average,		142.5	16.4		141.6	16.7
YEAR.		PSYCHOLOGY CLASS.				
1893	22	153.1	14.2	22	154.6	16.4
1894	30	163.3	19.5	42	151.4	17.4
1895	43	158.4	12.9	37	147.6	16.2
1896	53	155.8	14.6	33	155.6	16.3
1897	64	156.4	15.4	48	156.2	18.5
Average,		157.4	15.3		153.1	17.0

the constant error decreases. For the university students the variation is about three-fourths as large as for the fourth grade pupils, and is about one-half as large as the constant error.

The width of the bill is judged somewhat more accurately, especially by the university students. The accompanying figures indicate the nature and extent of the constant errors in both length and width. The dimensions of the bill used in these figures and also at the head of Tables VII and VIII were obtained by the actual measurement of fourteen five-dollar bills, ten two-dollar bills and ten one-dollar bills. The



Size of five-dollar bill. Average.—Heavy line indicates actual size of bill. Broken lines show the averages of all judgments in the several divisions.



Size of five-dollar bill. Maximum and minimum judgments.—Heavy line indicates actual size of bill. Broken lines show average of extreme cases. Each dimension is the average of ten per cent. of all judgments in that particular division.

TABLE VIII.

Width of Bill = 78.5 mm.

FOURTH GRADE.

AGE.	No. of Persons.	MALES.		No. of Persons.	FEMALES.	
		Average.	Av. Variation.		Average.	Av. Variation.
9	45	55.9	11.0	60	58.0	7.7
10	63	58.6	8.7	63	57.1	10.7
11	34	59.4	9.6	53	60.9	9.2
12	33	55.2	10.3	27	61.3	9.7
13	19	54.5	7.2	16	55.1	8.6
Average,		56.7	9.4		58.5	9.2
		EIGHTH GRADE.				
13	25	65.4	6.1	41	67.2	7.1
14	31	66.9	7.7	40	64.8	6.9
15	29	62.7	6.3	47	64.7	7.6
16	21	69.0	7.6	41	66.7	5.6
Average,		66.0	6.9		65.9	6.8
YEAR.		PSYCHOLOGY CLASS.				
1893	22	70.5	4.9	22	71.2	8.1
1894	30	73.4	9.5	42	72.8	7.1
1895	43	69.9	5.9	37	68.2	6.6
1896	53	69.0	5.6	33	69.1	5.2
1897	64	70.2	6.2	48	70.1	6.7
Average,		70.6	6.4		70.3	6.7

greatest variation in length of these bills was nine millimeters, and in width four millimeters. The bills were taken at random and the average dimension became the standard.

The explanation of so large constant errors will occur to every one. We seldom see bills spread out at full size. They are generally folded, and we observe one end as a mark of their denomination. If they were made of clay or steel we should probably exaggerate their size. Of course we are not accustomed to think of value in relation to size of bills, since all denominations are practically the same in size. The fact that the error is slightly less for the width than for the length adds to the force of this explanation. We are more accustomed

to see the width of a bill, and generally the width is seen in a way to enlarge our estimate of its size, inasmuch as it becomes the *length* of the folded bill.

I asked a few experts in banks to draw this figure and they made exceedingly small errors. One drew a rectangle 183x 72 mm. When asked how he estimated the size for his drawing, he replied that he imagined it passing through his fingers. Hence with him the judgment was rather tactual than visual. Another bank teller drew the length exactly right, but overestimated the width by ten millimeters.

TABLE IX.

Three-Inch Circle = 76.2 mm.

FOURTH GRADE.

AGE.	No. of Persons.	MALES.		No. of Persons.	FEMALES.	
		Average.	Av. Variation.		Average.	Av. Variation.
9	44	72.3	9.8	60	70.5	14.3
10	63	79.0	12.6	63	76.5	11.6
11	34	78.2	8.5	53	75.2	10.3
12	33	79.5	10.7	28	70.4	11.8
13	19	76.8	13.9	16	70.9	15.3
Average,		77.2	11.1		72.7	12.7

EIGHTH GRADE.

13	25	78.0	8.5	41	74.6	10.5
14	31	81.8	6.8	40	75.8	8.2
15	29	74.9	8.1	47	74.7	8.9
16	21	77.0	9.0	42	79.1	9.9
Average,		77.9	8.1		76.1	9.4

PSYCHOLOGY CLASS.

YEAR.						
1893	22	76.0	9.0	22	74.5	7.1
1894	30	85.0	10.7	42	77.4	8.8
1895	43	77.0	8.5	37	79.3	6.5
1896	53	80.7	9.6	33	78.2	7.7
1897	64	78.6	9.2	48	77.7	9.4
Average,		79.5	9.4		77.4	7.9

IX.

A Circle Three Inches in Diameter.

Draw a circle three inches in diameter (or three inches across), was the problem whose results are given in Table IX. The subjects were not prohibited from marking off a line as diameter, and were allowed to make corrections as usual. Perhaps nearly half used the method of a line as a guide, though I always suggested that they make the circle with freehand movement and attend to size more than to roundness.

The boys of all grades made the circles too large, but the error of the lowest grade is very small. The girls of the lowest grade made the circles too small. The higher grades make larger circles, and again the eighth grade children are more nearly right than the university students. As far as this problem alone is concerned, it would have been more satisfactory to have asked for a line three inches long. I wished, however, to use the circle for another purpose involving area, and besides, this whole investigation is on estimation of surface. While Table IX does not add much to the results of the preceding tables, it corroborates our conclusions drawn from those tables.

The variation of the girls in the fourth grade is much greater than that of the boys. In the eighth grade it is still somewhat greater, but in the university classes the girls now differ from each other less than the boys do. The subordinate groups in each division differ from each other more than in any other table except that of the square. Since this difference is not influenced by age, its cause is probably to be found in the greater difficulty of the problem.

X.

A Square One Inch on a Side.

A square inch is 25.4 millimeters on a side. There is a slight tendency in all classes, except university women, to make it too large. The average of the university women is almost exactly the right size. The variation is not large, but the more advanced students differ from each other nearly as much as the elementary pupils of the fourth grade.

This problem is unlike that of the reproduction of coins. It depends upon the constructive imagination rather than upon a concrete memory image. We are accustomed to see coins as surfaces, while the square inch is, perhaps, never seen by the average person with full recognition of its name

TABLE X.

Square Inch = 25.4 mm.

FOURTH GRADE.

AGE.	No. of Persons.	MALES.		No. of Persons.	FEMALES.	
		Average.	Av. Variation.		Average.	Av. Variation.
9	43	25.5	3.0	58	27.8	4.9
10	63	26.0	3.5	63	27.8	4.7
11	32	26.8	2.8	53	27.6	4.0
12	32	27.6	3.9	27	29.6	4.8
13	18	25.4	5.5	16	29.4	4.0
Average,		26.2	3.7		28.4	4.5

EIGHTH GRADE.

13	25	25.6	2.4	41	27.4	3.9
14	31	26.6	2.9	40	26.6	2.7
15	29	24.4	2.4	47	27.3	3.1
16	21	26.4	3.5	42	27.4	3.6
Average,		25.8	2.8		27.2	3.3

YEAR.	PSYCHOLOGY CLASS.					
1893	22	25.6	3.2	22	24.6	2.5
1894	30	27.5	4.2	42	25.8	2.8
1895	43	25.8	3.3	37	26 0	2.9
1896	53	26.6	3.9	33	25.3	3.4
1897	64	25.6	3.7	48	25.1	3.6
Average,		26.2	3.7		25.4	3.0

and with confidence in the accuracy of its dimensions. We have no article in common use which represents to us the surface of a square inch. Our visual image of this figure, then, is constructed from our memory image of a linear inch. The table shows that there is a slight constant tendency to overestimate the area of this square, and that the tendency is more decided with the lower grades, especially on the part of the girls. In all judgments of this kind, even more than in such as are represented in the first five tables, the direction of the error is a function of the mental character, and usually it may be predicted with considerable accuracy.

XI.

An Equilateral Triangle Equal in Area to the Three-Inch Circle and the One-Inch Square.

This problem was not given to the children of the fourth grade, because it was not believed they would be able to understand exactly what was wanted, without more suggestion than it was thought best to give. The eighth grade and university classes tried to solve the problem, but with rather unsatisfactory results, as the table shows. Nothing particularly important was expected from this exercise. It was suggested

TABLE XI.

Equilateral Triangle = 110 mm. on a side.

EIGHTH GRADE.

AGE.	No. of Persons.	MALES.		No. of Persons.	FEMALES.	
		Average.	Av. Variation.		Average.	Av. Variation.
13	25	110.0	15.9	40	108.2	20.4
14	31	113.6	14.6	40	107.7	16.7
15	29	107.2	15.7	47	102.8	18.6
16	21	102.4	15.1	42	102.7	13.6
Average,		108.3	15.3		105.3	17.3
YEAR.		PSYCHOLOGY CLASS.				
1893	22	108.4	14.1	22	103.3	9.2
1894	29	118.0	17.3	40	104.2	13.4
1895	43	110.5	17.1	37	109.8	14.3
1896	53	112.4	16.0	33	105.7	15.1
1897	64	109.2	18.4	48	107.2	16.4
Average,		111.7	16.6		106.0	13.7

by the previous problem of squaring the circles, and while the results are not as surprising as those in Table VI, they are worth noting. The constant error is insignificant, though the variation is larger than in most other tables. There is considerable uniformity in the results of both classes and of the groups in each class. It is evidently a much simpler matter to include the large circle and the small square in an equilateral triangle, than to reduce five diverse circles to the form of a square.

The average length of the three sides was taken as the

standard in order to save computation. Only in a few cases would the area differ greatly from an equilateral triangle with sides equal to the average of those drawn. The figures at the head of the table indicate the size of an equilateral triangle, which is equivalent to a three-inch circle and an inch square. The small constant errors made in drawing the circle and square must therefore be considered in calculating the constant error for the triangle.

The Dimensions of the Paper.

Tables XII, XIII and XIV contain the estimates on the length and width of the paper, and on the length of the diagonal. The estimates are, of course, in inches.

TABLE XII.

Length of Paper = 14 inches.

FOURTH GRADE.

AGE.	No. of Persons.	MALES.		No. of Persons.	FEMALES.	
		Average.	Av. Variation.		Average.	Av. Variation.
9	45	13.9	1.8	57	13.4	1.4
10	63	14.9	2.3	63	13.8	1.9
11	35	14.5	2.0	53	13.8	1.9
12	33	13.9	1.4	28	12.9	.9
13	17	14.5	1.6	16	12.7	1.6
Average,		14.3	1.8		13.3	1.5
		EIGHTH GRADE.				
13	25	14.1	1.0	41	14.2	1.2
14	31	13.7	1.5	40	14.1	1.4
15	28	14.5	1.3	46	13.7	1.2
16	21	13.7	1.3	42	13.6	1.4
Average,		14.0	1.3		13.9	1.3
YEAR.		PSYCHOLOGY CLASS.				
1893	22	15.2	1.3	22	15.1	1.3
1894	29	14.0	1.5	42	14.4	1.8
1895	43	14.7	1.2	37	14.3	1.6
1896	48	14.5	1.6	29	14.4	1.4
1897	64	13.9	.9	48	14.1	1.8
Average,		14.5	1.3		14.5	1.6

TABLE XIII.

Width of Paper = 9 inches.

FOURTH GRADE.

AGE.	No. of Persons.	MALES.		No. of Persons.	FEMALES.	
		Average.	Av. Variation.		Average.	Av. Variation.
9	44	8.7	1.5	55	8.6	1.2
10	61	9.2	1.6	61	8.9	1.6
11	34	8.7	1.3	53	8.7	1.4
12	33	9.5	1.5	27	8.2	1.0
13	17	9.2	1.3	16	8.2	1.5
Average,		9.1	1.4		8.5	1.3
		EIGHTH GRADE.				
13	25	8.8	.9	41	9.0	.8
14	31	8.5	1.0	40	9.0	1.3
15	28	9.0	.8	46	8.8	1.0
16	21	8.5	1.1	42	8.7	1.0
Average,		8.7	1.0		8.9	1.0
YEAR.		PSYCHOLOGY CLASS.				
1893	22	9.8	.9	22	9.6	1.0
1894	29	9.1	1.4	42	9.4	1.2
1895	43	9.4	1.2	37	9.2	1.2
1896	48	9.5	1.2	29	9.6	1.2
1897	64	9.1	1.1	48	9.3	1.2
Average,		9.4	1.2		9.4	1.2

A slight tendency to overestimate the length is seen in all divisions except that of the fourth grade girls, who underestimate this dimension about three-quarters of an inch. The variation is the lowest we have yet found, being on the average somewhat less than ten per cent. of the estimated size of the paper. Most children in the public schools have a fair conception of the foot as a unit of length. The paper being appreciably longer than a foot would be comparatively easy to estimate. There are, however, individuals who have no conception of any unit of length, as the table of extreme cases shows (Table XV).

The width of the paper was not as accurately judged as

the length, even by the university students. I infer that the principal cause is the greater difference, both relative and absolute, between the standard — one foot — and the dimensions to be estimated. The girls of the lower class underestimate the width about as much as the students of the psychology class overestimate it.

TABLE XIV

Diagonal of Paper = 16.6 inches.

AGE.	No. of Persons.	MALES. Average.	MALES. Av. Variation.	No. of Persons.	FEMALES. Average.	FEMALES. Av. Variation.
FOURTH GRADE.						
9	41	17.6	2.9	52	17.0	2.5
10	60	18.7	3.2	60	17.2	2.7
11	30	17.5	3.1	49	17.3	3.3
12	31	17.7	2.7	27	16.3	1.8
13	15	18.8	3.4	16	16.4	3.5
Average,		18.1	3.1		16.8	2.8
EIGHTH GRADE.						
13	25	17.4	1.6	38	18.0	1.8
14	29	16.9	1.6	36	17.9	2.0
15	28	18.0	2.1	46	17.7	2.2
16	21	17.1	1.6	38	17.3	2.2
Average,		17.4	1.7		17.7	2.1
YEAR.		**PSYCHOLOGY CLASS.**				
1893	22	19.1	2.9	22	18.4	2.1
1894	29	17.4	2.5	40	18.2	2.1
1895	43	18.3	1.9	37	17.4	2.1
1896	48	17.5	1.8	28	17.9	2.1
1897	64	17.1	1.3	48	17.1	2.2
Average,		17.9	2.1		17.8	2.1

The diagonal of the paper was still more difficult to judge, partly because of its greater length and the absence of any distinct line like the edge of the paper, and partly because it differed more from the most common unit of linear measure, the foot. The actual length of the diagonal is 16.6 inches.

It is seen from the table that neither age nor academic rank aids one much in this exercise. The length of the diagonal is overestimated by nearly every division. The girls of the fourth grade, having the lowest figures here, as in the two preceding tables, have overestimated least, and hence seem to be most accurate in judgment of this dimension.

The variation naturally rises with the increased difficulty of the problem. It also differs more than usual in the various divisions of the same class of observers; as, for example, in the groups of twelve and thirteen year old girls of the fourth grade, where it varies from 1.81 to 3.47, and in the men of the university classes from 1.26 to 2.90. On the other hand the women of the university classes have the most uniform variation of any table.

XV.

Maximum and Minimum Judgments.

Some notion of the range of judgment is a necessary supplement to the foregoing exhibit. It has seemed to me best to present this in a condensed table containing a certain proportion of the judgments of each class. I have, therefore, given in Table XV the average of ten per cent. of all judgments in each class selected from the highest figures, and also the average of ten per cent. taken from the lowest figures for each problem. Table XV then gives for every hundred judgments the average of the ten highest and the average of the ten lowest. For comparison with my other tables this seems to be a better method than to give the judgment which is exceeded by ten per cent. and that which is not reached by ten per cent. of the subjects.

The table explains itself. In the fourth grade the average of the highest judgments is almost exactly twice the average of the lowest judgments. With age and learning the difference between extremes becomes less; but even with university students the average of the maximum is about one and two-thirds times the average of the minimum judgments.

The first problem of special interest in this table is the maximum size of the dime. The average of the tenth yielding the largest judgments barely exceeds the real size of the dime, while the average of the smallest judgments is almost incredibly small. Many of the children drew a dime smaller than the diameter of a common lead pencil, and only about as many made it as large as a nickel. Not one person in ten made it large enough, while almost one in ten made it only half big enough.

The most interesting problem is the five-dollar bill. The

TABLE XV.

Maximum and Minimum Judgments.

Each number is the average of ten per cent. of the persons in that particular division.

	Size. Mm.	SEX.	4TH GRADE.		8TH GRADE.		PSY. CLASS.	
			Larg't	Small't	Larg't	Small't	Larg't	Small't
Dollar,	37.8	M	49.5	26.9	46.1	31.1	48.8	30.7
		F	52.1	24.1	48.8	31.4	49.9	31.6
Half Dollar,	30.6	M	40.3	20.7	38.0	24.2	40.0	24.5
		F	42.7	17.9	39.6	24.9	40.6	25.3
Quarter Dollar,	24	M	31.0	15.9	29.0	17.9	31.8	18.6
		F	32.8	14.5	30.8	19.2	31.2	19.5
Dime,	19	M	20.0	9.9	18.7	11.5	20.3	11.0
		F	19.7	9.7	19.1	11.6	19.5	11.8
Nickel,	21	M	22.7	12.1	21.8	13.9	24.4	13.8
		F	22.8	10.9	22.4	13.4	23.7	15.4
Square,	54.2	M	137.7	45.0	93.0	43.4	82.7	42.6
		F	133.1	43.2	114.5	42.6	94.7	45.0
Bill, long,	186.5	M	167.3	80.2	181 4	107.9	191.8	122.9
		F	165.7	81.5	178.7	103.9	188.4	111.9
Bill, wide,	78.5	M	78.1	37.3	80.1	49.5	85.5	56.0
		F	77.3	38.0	79.5	49.6	85.2	54.3
Circle, 3 in.,	76.2	M	104.1	51.3	94.4	59.7	105.1	60.5
		F	102.7	44.4	99.0	54.3	96.8	59.6
Square Inch,	25.4	M	36.7	18.2	32.7	19.8	34.9	18.6
		F	41.3	20.5	35.4	20.9	32.4	18.5
Triangle,	110	M			149.0	77.1	152.1	78.9
		F			150.2	67.8	139.2	76.8
	Inches.							
Paper, long,	14	M	20.7	11.3	17.3	10.9	18.0	11.6
		F	18.1	10.0	17.0	11.0	18.6	11.2
Paper, wide,	9	M	12.7	5.9	11.0	6.3	11.7	7.2
		F	12.1	5.9	11.8	6.8	12.3	7.8
Paper, diag.,	16.6	M	26.4	12.8	22.4	13.8	23.0	14.3
		F	23.7	11.4	23.4	13.7	23.3	13.8

accompanying figures illustrate the extremes and also the average judgments. In the fourth and also in the eighth

grade, the maximum judgment of length fails to reach the actual length, and even in the university classes exceeds it only a few millimeters. The minimum of the fourth grade gives us a bill scarcely longer than the width of a real bill and a width about one-fifth as great as the real is long. Hence, the average minimum bill is about one-fifth the actual bill. Remembering that all maxima and minima are each the average of ten per cent. of the judgments of the given division, the monstrosity of the results is apparent. Those who make the bill large enough are veritable freaks; while those who make it only half large enough are the normal persons.

SUMMARY.

From observations on nearly eleven hundred persons, the following points, among others, may fairly be said to be established:

I. Our notions of the size of familiar objects differ widely.

II. Young children underestimate the size of coins and bills.

III. Mature persons of intelligence overestimate the size of the silver dollar, half dollar and quarter dollar.

IV. All classes of persons underestimate the size of the dime, nickel and bill.

V. Girls make larger coins than boys and also larger equivalent squares. In other problems of this investigation the boys make the larger figures.

VI. The judgments of the eighth grade children are more uniform than those of either the fourth grade children or university students, and are more nearly like the latter than the former.

VII. The more advanced classes produce *larger* coins and bills than the elementary classes; but within the same class age causes no appreciable effect.

VIII. It is probable that the reproduction of such objects as are here studied, is a function of the *personality* apart from knowledge or mental acumen.

IX. The enormous errors in reproducing the bill and dime, suggest that the teaching of drawing in public schools may profitably concern itself more with size.

X. To young children a memory image is smaller than its object, while in the minds of adults it may exceed the object in size.